Chris Elliott – Non-Technical Research Statement

My research studies applications of quantum field theory – a theory in physics which models the behaviour of subatomic particles – to mathematics, using modern mathematical tools. Physicists studying quantum field theory often describe aspects of these physical theories using mathematical techniques. In particular, in the last few decades physicists have introduced many ideas from modern geometry and representation theory (the mathematical study of symmetry) into their discipline; we often say that a physical theory is "modelled" by an abstract mathematical object. Because physicists aim to answer qualitative and quantitative questions about physical systems, these models are not constructed at a mathematical level of rigour.

In contrast, my work aims to rigorously construct mathematical models for physical theories. This leads to new insights in mathematics because it allows us to translate physical ideas and intuitions into new mathematical results. As an example, there are many situations in physics where two different kinds of quantum field theory are believed to model the same phenomena: just like having two descriptions of the same theory. Pairs of theories like this are said to be related by a "duality". If we can construct mathematical models for these physical theories, a duality tells us that the two mathematical objects modelling the two physical theories are actually equivalent, which is often a new and surprising mathematical result. By mathematically modelling physical theories one therefore obtains mathematical conjectures – statements which are not yet proven – and further, one can apply ones physical intuition about how the duality works to get ideas about how to prove the conjectures, obtaining a new and unexpected theorem. Even if the duality cannot be made into a rigorous mathematical argument, this translation offers a new perspective on the mathematical objects in question, and may suggest novel approaches to mathematical questions.

New connections between fields, such as this kind of application of physics to mathematics, can be very profitable. Any new connection allows a translation of both the concrete tools and the more abstract intuitions of one field to the other, allowing for new ways of thinking about or approaching a problem. These new approaches not only allow us to solve previously intractable problems, but also broaden and deepen our understanding of a field, suggesting new questions based on a fusion of the two perspectives. This is true not only between different disciplines such as physics and mathematics, but also between different areas of the same discipline. For example, many of the questions I study were originally inspired by questions in number theory, and new developments in my field have led to, and will continue to lead to, new answers to number theoretic questions.

One example of this kind of interdisciplinary link appears in an influential piece of work performed by the physicists Edward Witten and Anton Kapustin. They proposed a mathematical description of certain physical theories which are expected to be in duality, although they did not intend to provide mathematically rigorous constructions. These mathematical descriptions had been conjectured to be equivalent before a link to physics was ever discovered, this conjectural equivalence is an unproven conjecture in mathematics known as the "geometric Langlands conjecture"; the central conjecture in its mathematical field.

In a paper written jointly with Philsang Yoo I provided a rigorous mathematical model for aspects of the theories Kapustin and Witten described. We are currently working on extending this work in several ways. An important deficiency of Kapustin and Witten's proposal is that their mathematical descriptions do not quite match up with what is required for the geometric Langlands conjecture, they instead describe an older form of the geometric Langlands conjecture which is known to be false. We have a physically motivated proposal that corrects Kapustin and Witten's description, lining up the mathematical models with a corrected form of the conjecture. Our proposal also allows us to make a new, broader conjecture motivated by our physical description, which is likely to suggest new avenues along which we can approach the geometric Langlands conjecture and other related problems.

I also intend to extend this work to provide rigorous models for a larger range of theories called "theories of class S", that have been studied in the physics literature by Davide Gaiotto. Gaiotto proposed a complex collection of dualities between these theories of class S, incorporating and extending the duality studied by Kapustin and Witten. By rigorously modelling these theories, I will be able to make new mathematical conjectures based on these dualities. We expect these conjectures to provide new links to other areas of study in geometry, broadening our outlook on the field.

I am also working on a mathematical approach to a different type of theory in physics: theories describing gravity. There are mathematical models for gravity which make sense in any number of dimensions, not just the familiar situation of three spacial dimensions plus one time dimension. These mathematical models can also be made to incorporate what's called "supersymmetry". This is a type of structure a mathematical model can have, which is important for many speculative theories physicists have introduced to solve problems in present day models for subatomic particles. Supersymmetry is also important from a mathematical perspective because it allows us to simplify a complicated physical theory, replacing it by a theory that is easier to model but that nevertheless captures some facets of the full physical theory; we call this simplification procedure "twisting". In my work with Philsang Yoo I gave a mathematical definition of twisting which played an essential role in our model for the Kapustin-Witten theories. I intend to use this construction to give a mathematical model for theories of gravity, which will allow us to investigate the mathematical consequences of dualities involving theories of gravity. Preliminary work implies that we will obtain particularly interesting mathematical theories when we model theories of supersymmetric gravity in eleven dimensions, because the behaviour of supersymmetric theories in this dimension suggest tantalizing links to topics in differential geometry.

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