Math 132H – Homework 1 – Introduction to Integration

Due: Wednesday September 2nd

You should explain your reasoning carefully using English sentences where appropriate, not only equations. You may use the textbook and your notes, and you're welcome to discuss the problems with one another, with me, and with the TA, but your final answers should be your own and in your own words

1. (a) Let $f(x) = \int_0^{\pi/2} (\sin(u) - x)^2 \mathrm{d}u$. *Without computing the integral*, find x so that $f(x)$ is minimal. (Hint: do you remember how to find the maxima and minima of a function? If not, review §4.1 in the textbook.)

   (b) Now fix any $a < b$ and choose any continuous function $g(u)$. For which value of x is the expression $\int_a^b (g(u) - x)^2 \mathrm{d}u$ smallest? Give your answer in terms of a, b and $g(u)$.

2. Recall that if f and g are two continuous functions, and $f(x) \leq g(x)$ for all x in the interval $[a, b]$, then

$$\int_a^b f(x)\mathrm{d}x \leq \int_a^b g(x)\mathrm{d}x.$$

   (a) Use this fact to show that if $f(x)$ has maximum value M and minimum value m on the interval $[a, b]$ then

$$m(b-a) \leq \int_a^b f(x)\mathrm{d}x \leq M(b-a).$$

   (b) Hence show that

$$2 \leq \int_1^3 \frac{2x^2}{1+x^2}\mathrm{d}x \leq \frac{18}{5}.$$