Math 132H – Homework 2

Due: Wednesday September 16th

You should explain your reasoning carefully using English sentences where appropriate, not only equations. You may use the textbook and your notes, and you're welcome to discuss the problems with one another, with me, and with the TA, but your final answers should be your own and in your own words

- 1. Consider the region contained between the curve $y = \frac{x}{\sqrt{x^2-5}}$, the line x = 5, the line $y = \sqrt{2}$, the x axis and the y-axis.
 - (a) Sketch the region in question, computing the points where the boundary curves intersect, and indicating any vertical and horizontal asymptotes.
 - (b) For which slope m does the line y = m(x 5) cut this region exactly in half?
- 2. This question is about the volume of barrels, which was a topic of active study in 17th century mathematics. For example, Johannes Kepler, most famous today for his laws of planetary motion, wrote a treatise in 1615 entitled *Nova Steriometria Doliorum Vinariorum* concerning the volumes of barrels ¹.

In this question, a *barrel* is considered to have a circular cross-section, and curved sides described by parabolas. Consider a barrel of height h, and suppose that the smallest radius of the cross-sectional circles is r, at the top and bottom of the barrels, and that the largest radius of the cross-sectional circles is R, occuring at the middle of the barrel.

- (a) Set up an equation f(x) = y describing the parabolic curve along the side of the barrel given the specified radii (so f(x) will be a quadratic function). Hint: it'll be easier if you place the two ends of the barrel at x = -h/2 and x = h/2.
- (b) Find a formula for the volume of the barrel in terms of h, r and R.
- (c) A typical barrel has a volume of 225 liters. Suppose that $R = 0.35 \times h$ and that $r = 0.25 \times h$. Find h, r and R for the given volume in metres (recall that a liter is $0.001m^3$). Give your answers to 2 decimal places.

¹You can find it in full here: http://posner.library.cmu.edu/Posner/books/pages.cgi?call=520_K38PN&layout=vol0/part0/copy0& file=0001.