Math 132H - Homework 3

Due: Wednesday September 30th

You should explain your reasoning carefully using English sentences where appropriate, not only equations. You may use the textbook and your notes, and you're welcome to discuss the problems with one another, with me, and with the TA, but your final answers should be your own and in your own words

In these exercises we'll learn about a method called the *Weierstrass substitution*, that can be used to integrate any ratio of sums of trigonometric functions by transforming them into rational functions. The Weierstrass substitution is t = tan(x/2) (it's sometimes just called the tangent half-angle substitution).

1. Show that

$$\sin(x/2) = \frac{t}{\sqrt{1+t^2}}$$
 and $\cos(x/2) = \frac{1}{\sqrt{1+t^2}}$,

and hence work out expressions for sin(x) and cos(x) in terms of the new variable t (using double angle formulae).

- 2. Write dx in terms of the variable t.
- 3. Consider the integral

$$\int \frac{\mathrm{d}x}{a\sin(x) + b\cos(x) + c}$$

where a, b, c are real numbers and $b \neq c$. Let $D = a^2 + b^2 - c^2$. Use the Weierstrass substitution to evaluate the integral in each of the following three situations:

- (a) D > 0.
- (b) D < 0.
- (c) D = 0.
- 4. Use the Weierstrass substitution and the method of partial fractions to compute

$$\int \frac{\cot^2(x) + \csc^2(x)}{\cot(x)\csc(x) + \frac{3}{4}\csc(x)} \mathrm{d}x.$$