

Math 211 – Multivariate Calculus – Homework 10

Due: Friday December 2nd

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

1. Consider the vector field

$$\mathbf{F}(x, y, z) = (2x - y - z, x + y - z^2, 3x - 2y + 4z).$$

Find the work done in moving a particle in a force field given by \mathbf{F} along the following curves.

- (a) C_1 , a straight line from the origin to $(1, 1, 1)$.
 - (b) C_2 , a circle around the origin in the xy -plane of radius 3, traversed anticlockwise.
2. Consider the curve C given by $\mathbf{r}(t) = (t^2, 2t, t^3)$ from $t = 0$ to 1.
- (a) Let $f(x, y, z) = 2xyz^2$. Evaluate $\int_C f(x, y, z) d\mathbf{r}$. Note that your answer should be a vector.
 - (b) Let $\mathbf{F}(x, y, z) = (xy, -z, x^2)$. Evaluate $\int_C \mathbf{F}(x, y, z) \times d\mathbf{r}$. Again, note that your answer will be a vector.
3. Evaluate the line integral

$$\int_C (10x^4 - 2xy^3)dx - 3x^2y^2dy$$

where C is the curve in the xy -plane given by the equation $x^4 - 6xy^2 = 4y^2$, from the point $(0, 0)$ to the point $(2, 1)$.

4. Let $f = \sin(x - 2y)$, and let $\mathbf{F} = \nabla f$.
- (a) Sketch the vector field \mathbf{F} in the xy -plane.
 - (b) Find a curve C_1 that is not closed with the property that $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 0$.
 - (c) Find a curve C_2 with the property that $\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 1$.