Math 211 – Multivariate Calculus – Homework 7

Due: Friday October 28th

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

1. The method of Lagrange multipliers assumes that minimum and maximum values exist, but this is not always the case, because the region R given by the constraint may not be bounded.
   (a) Use Lagrange multipliers to find the minimum value of $f(x, y) = x^2 + y^2$ subject to the constraint $xy = 2$.
   (b) Show that $f(x, y)$ does not have a maximum value on the region given by the constraint.
2. Use the method of Lagrange multipliers to show that the triangle with perimeter p with the highest area is an equilateral triangle. You may use *Heron's formula* for the area of a triangle with perimeter p and side lengths a, b, c:
$$\mathrm{Area} = \sqrt{p/2(p/2-a)(p/2-b)(p/2-c)}.$$
3. Compute
$$\int_{-1}^{2}\int_{0}^{4}(1-xy^2)\mathrm{d}x\mathrm{d}y$$
   as a limit of Riemann sums. You may use the sums
$$\sum_{j=1}^{n} j = \frac{1}{2}n(n+1)$$
$$\text{and } \sum_{j=1}^{n} j^2 = \frac{1}{6}n(n+1)(2n+1).$$
4. Find the average value of the function $f(x, y)$ on the rectangle R (compute the integral and divide the result by the area of the rectangle).
   (a) $f(x, y) = xy^2$, $R = [0, 6] \times [-1, 1]$.
   (b) $f(x, y) = e^y\sqrt{e^y + x}$, $R = [0, 2] \times [0, 1]$.