Math 211 – Multivariate Calculus – Homework 9

Due: Friday November 18th

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

1. Use the given change of variables to evaluate each of the following integrals
   (a) $\iint_R (x-3y)\mathrm{d}x\mathrm{d}y$, where R is the triangle with vertices $(0,0),(2,1),(1,2)$. Use the change of variables $x=2u+v, y=u+2v$.
   (b) $\iint_R xy\mathrm{d}x\mathrm{d}y$, where R is the region in the first quadrant bounded by the straight lines $y=x, y=3x$ and the hyperbolae $xy=1$ and $xy=3$. Use the change of variables $x=u/v, y=v$.
2. Sketch the solid region whose volume is represented by each of the following integrals, then compute the given volumes.
   (a)
   $$\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho^2\sin(\phi)\mathrm{d}\rho\mathrm{d}\theta\mathrm{d}\phi.$$
   (b)
   $$\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec(\phi)} \rho^2\sin(\phi)\mathrm{d}\rho\mathrm{d}\theta\mathrm{d}\phi.$$
3. (a) Use cylindrical coordinates to find the volume of the region R between the paraboloid $z=24-x^2-y^2$ and the cone $z=2\sqrt{x^2+y^2}$.
   (b) Find the center of mass of the region R, assuming it has constant density.
4. (*Paraboloidal coordinates*.) Consider the change of coordinates $x=uv\cos(\theta), y=uv\sin(\theta), z=\frac{1}{2}(u^2-v^2)$.
   (a) Sketch the surfaces $u=1, v=1$ and $\theta=\pi/2$ in the xyz plane.
   (b) Compute the Jacobian for this change of coordinates.
   (c) Use these coordinates to evaluate the integral
   $$\iiint_R xz+yz\mathrm{d}x\mathrm{d}y\mathrm{d}z$$
   where R is the region between the two paraboloids $2z=1-x^2-y^2$ and $2z=-1+x^2+y^2$, and with $y\geq 0$.