Math 250 – Number Theory – Homework 10

Due: Friday May 5th

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

1. In each of the following cases, determine the number of square roots of s modulo n.

   (a) $s = 96$, $n = 97$.

   (b) $s = 11$, $n = 70$.

   (c) $s = 6$ $n = 37$.

2. Find all solutions to the following quadratic congruences.

   (a) $9x^2 - 6x - 4 \equiv 0 \mod 17$.

   (b) $x^2 + 22x + 122 \equiv 0 \mod 61$.

   (c) $x^2 + 12x + 36 \equiv 1 \mod 85$.

3. (a) Let n be a natural number. Prove that if $[a] \in Q_n$ then a is not a primitive root modulo n.

   (b) Show that 12 is not a primitive root modulo 109.

4. (a) Let $n = n_1 n_2$ where n_1, n_2 are coprime natural numbers. Prove that a is a quadratic residue modulo n if and only if it is a quadratic residue modulo both n_1 and n_2

   (b) Is 61 a quadratic residue modulo 77?