Math 250 – Number Theory – Homework 2

Due: Friday February 17th

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

- 1. Determine the set of integer values of c for which the following equations admit integer solutions. Find a specific solution for the smallest positive value of c in this set.
 - (a) c = 550x + 891y.(b) c = 625x - 648y.(c)

$$c = 910x + 4420y + 2353z.$$

- 2. (a) Prove that if d is a common divisor of two non-zero integers a and b then $d|\operatorname{gcd}(a,b)$.
 - (b) Prove that if m is a common multiple of two non-zero integers a and b then lcm(a, b)|m.
- 3. (a) The equation

$$\operatorname{lcm}(a, b, c) = \frac{abc}{\operatorname{gcd}(a, b, c)}$$

is not a correct way of computing the lcm of three numbers. Demonstrate this by finding a set of three positive integers a, b, c for which the above equation is false.

(b) Instead, prove that the equation

$$\operatorname{lcm}(a, b, c) = \frac{abc}{\gcd(ab, bc, ac)}$$

does hold for any three positive integers a, b, c.

- 4. (a) The numbers 3, 5, 7 are all prime. Do there ever exist integers p, p + 2, p + 4 that are all prime for any p > 3?
 - (b) There are four primes between 10 and 20. Are there ever four prime numbers between 10x and 10(x+1) for any integer x > 1?