## Math 250 – Number Theory – Homework 9

## Due: Friday April 28th

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

- 1. Find all solutions to the following congruences:
  - (a)  $x^3 \equiv 1 \mod 63$ .
  - (b)  $x^4 \equiv 110 \mod 115$ .
- 2. (a) Find a number a so that every element of  $U_{16}$  is congruent to either  $a^k$  or  $-a^k$  for some natural number k.
  - (b) Hence find all solutions to the congruence  $x^7 \equiv 11 \mod 16$ .
- 3. (a) Find all the elements of  $Q_{25}$ . For each element  $[a] \in Q_{25}$  compute its order, and give a list of all the square roots of [a] in  $U_{25}$ .
  - (b) Find all the elements of  $Q_{60}$ . For each element  $[a] \in Q_{60}$  compute its order, and give a list of all the square roots of [a] in  $U_{60}$ .
- 4. (a) Let n be a natural number. Prove that if [a] and [b] are quadratic residues mod n then so is [ab].
  - (b) Prove that if [a] is a quadratic residue, and  $[a^*]$  is the inverse of [a], then  $[a^*]$  is also a quadratic residue.