Math 271 – Linear Algebra – Homework 2

Due: Friday February 14th

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

1. Show that if V_1 is a subspace of V_2 and V_2 is a subspace of V_3, then V_1 is a subspace of V_3.
2. Consider the following subsets of the vector space $F(\mathbb{R})$ of all functions $\mathbb{R} \to \mathbb{R}$. In each case either prove that the subset is a subspace, or explain why it is not a subspace.
   (a) The set of all functions f with the property that $f(0) = 0$.
   (b) The set of all functions f with the property that $f(0) = 1$.
   (c) The set of all functions f with the property that $|f(x)| < 1$ for all x.
   (d) The set of all even functions, i.e. functions f with the property that $f(x) = f(-x)$ for all x.
3. Let V be a vector space, and let U_1 and U_2 be subspaces of V. Show that the union $U_1 \cup U_2$ is a subspace of V if and only if $U_1 \subseteq U_2$ or $U_2 \subseteq U_1$.
4. (a) Show that, in $\mathbb{R}^2$, every straight line passing through the origin is a subspace.
   (b) Suppose $\boldsymbol{v}$ and $\boldsymbol{w}$ are two non-zero vectors in $\mathbb{R}^2$, and suppose that $\boldsymbol{w}$ is *not* a scalar multiple of $\boldsymbol{v}$. Show that if U is a subspace of $\mathbb{R}^2$ containing both $\boldsymbol{v}$ and $\boldsymbol{w}$ then $U = \mathbb{R}^2$.
   (c) Hence show that the only subspaces of $\mathbb{R}^2$ are $\{0\}$, the straight lines through the origin, and $\mathbb{R}^2$ itself.