Math 350 – Groups, Rings and Fields – Homework 6 (§10 and start of 11)

Due: Friday March 29th at 5pm

Please explain your answers carefully using full sentences, not only symbols. You may use the textbook and your notes, and you're welcome to discuss the problems with one another or with me. However, your final answers should be written on your own and in your own words.

At the top of the first page, please list any classmates you collaborated with while working on these exercises (so that we know to expect similar solutions).

A

1. Let $G = A_4$, and let $H = \langle(1\,2\,3)\rangle$. Describe the decomposition of G into a union of left cosets for the subgroup H. Then describe the decomposition of G into a union of right cosets for H.
2. Let G be the group Q of unit quaternions (if you don't remember the definition, it's on pages 47–48 in the textbook).
   (a) Describe a subgroup $H \subseteq G$ with four elements.
   (b) Describe the right cosets of H in G.
3. Let $G = A_4$. Find a normal subgroup H of G, and a normal subgroup K of H, with the property that K is not normal as a subgroup of G.
4. List all the subgroups of the group D_4. Which subgroups are normal, and which are not normal?

B

5. Let p and q be prime numbers, and let G be a group with pq elements. Prove that every proper subgroup of G (that is, every subgroup H of G with $H \neq G$) is cyclic.
6. Show that any group G of order 10 is either isomorphic to $\mathbb{Z}/10$ or to D_5. You may assume that G always contains an element of order 5.
7. (a) Prove that every group with 91 elements must have either an element of order 7 or an element of order 13.
   (b) $(*)$ (*Optional*:) Prove that every group with 91 elements must have an element of order 7 *and* an element of order 13 (if you attempt this problem you should read the last part of §10 of the textbook, about the *class formula*).