

Math 405 – Lie Groups and Lie Algebras – Syllabus

Spring 2025

- **Instructor:** Chris Elliott (pronouns: he/him)
You can reach me by e-mail at cellriott@amherst.edu. Please feel free to call me Chris.
- **My Office Hours:** Mondays 11:00 – 12:00, Wednesdays 11:00 to 12:40 and Fridays 12:30 to 13:30 in Chapin Hall room 018. I can also meet by appointment if none of those times work for your schedule.

What We'll Cover

In this course we'll be learning about the area of mathematics where group theory, linear algebra and geometry all meet. We'll learn about

- *Lie groups*: groups of linear maps.
- *Representations* of Lie groups: the ways a Lie group can act on a vector space by linear transformations – these appear in applications everywhere linear algebra appears (in other words, in every field of mathematics and most of the rest of the sciences).
- *Lie algebras*: like everywhere else in mathematics we make a complicated structure simpler by “differentiating” it: by taking a linear approximation. The “derivative” of a Lie group is known as a Lie algebra, and we'll learn all about their properties.

By the end of the course we will be able to discuss some of the great achievements of 20th century mathematics:

- The *structure* of Lie groups / algebras out of fundamental building blocks known as *simple* Lie groups / algebras.
- The *classification* of simple Lie groups / Lie algebras in terms of a few elementary families.
- The classification of *representations* of simple Lie groups / algebras in terms of a combinatorial object called the *highest weight*.

Schedule

We will meet three times a week for classes:

Monday, Wednesday and Friday at 2pm in **Seeley Mudd room 204**.

There will be one in-class midterm exam held on the following day:

- **Midterm:** Friday March 28th during class.

The midterm will be in person, and will be closed book. Calculators will not be allowed.

You will have the *choice* whether to take a final exam or a final project. You will have until *March 14th* to make your choice.

- **Final exam option:** Take-home final exam during finals week (exact date to be confirmed). You will have 24 hours to complete the exam, although it will be designed to be doable in 3 hours. The exam will be open note and open textbook, but no other resources may be used.
- **Final project option:** Individual written final project to be submitted by *Tuesday May 6th*. Project guidance and a list of possible topics will be shared early in the semester (you may also choose topics not on the list with instructor approval). You will also give a one hour oral presentation/Q&A on your project (one-on-one, not in front of the group) to be scheduled during finals week.

Makeup Exams

If you cannot make one of the exam times, please let me know as long as possible in advance and **at least two weeks beforehand**. I can arrange make-up exams for legitimate conflicts (e.g. for academic commitments, religious observances) but two weeks advance notice is necessary.

Textbooks

We will use the following two textbooks:

1. *Naive Lie Theory* by Stillwell (Undergraduate Texts in Mathematics, ISBN: 978-0-387-78214-0). We'll use this during the first part of the class where we study Lie groups, around the first 5 weeks.
2. *Introduction to Lie Algebras* by Erdmann and Wildon (Springer Undergraduate Mathematics series, ISBN: 978-1-84628-040-5). We'll use this book during the second part of the class where we study Lie algebras from around week 6 onwards.

I will assign optional but recommended reading from these books at the end of each class.

Homework

Homework will be assigned each week and due on **Fridays at 5pm**. There will not be homework due during the week with a midterm exam. The first homework will be due on Friday February 7th.

Homeworks will consist of 4–8 problems on the material we learned in the past week. Some problems that are particularly challenging will be marked with a (*). These problems are **optional**; you can earn 100% on the homework without attempting these, but good solutions will be worth extra credit!

You are encouraged to work on the homework in groups; this is often one of the best ways of learning. However, your final solution **must** be your own work; you should write up your answers on your own, without anyone else's work present (in other words, do not copy!). On the first page of your homework submission, please list the people that you worked with.

You will submit your homework online through **Gradescope** (<https://www.gradescope.com>). You should sign up for a free account using your Amherst College email address. Once you've created an account you should join the section using the following course code:

Gradescope course code: **PYV6Y4**.

Homework Extensions

I know that sometimes things come up that make it difficult to complete homework on time. As such, I will grant up to two homework extensions per person during the semester (you don't need to give a reason, just ask). If you want an extension on one of the homeworks, please e-mail me no later than the **day before** the homework is due.

Assessment Structure

Your grade will be calculated as follows.

- Homeworks: 40% (lowest score dropped)
- Midterm exam: 25%
- Final exam or project: 35%.

Accessibility

As the instructor of this course, I endeavor to provide an inclusive learning environment. However, if you experience barriers to learning in this course, let's connect to discuss ways to best support your access. If you have disability-related circumstances and are seeking academic accommodations (e.g. extra-time testing, reduced distraction test area, short breaks as needed, note taking assistance, etc.), Accessibility Services is eager to assist with identifying reasonable accommodations for the course. They can be contacted at accessibility@amherst.edu.

Honor Code

The Amherst College honor code applies to this course. All the work you submit, both for the exams and the homework, must be entirely your own. In particular, although discussing the homework in groups is encouraged, when you write down your solutions you should not be looking at anyone else's work. Copying somebody else's work is a violation of the honor code.

If you feel stuck or lost in the course, please get in touch with me either by e-mail or in office hours as early as possible. I will be happy to help you!