Homology and Cohomology – Week 4 Exercises

Read sections 6.1–6.3 of Armstrong's *Basic Topology* and sections 8.1 to 8.2 of tom Dieck's *Algebraic Topology* (also read the definition of "colimit topology" on page 10–11).

Think about these exercises, and write up solutions to three of them.

1. (a) Describe a triangulation of S^n for any $n \geq 0$.
   (b) Describe a triangulation of $\mathbb{RP}^n$ for any $n \geq 0$.
2. (Armstrong's exercise 8 on page 124) Show that the 'dunce hat' (Fig. 5.11) is triangulable, but that the 'comb space' (Fig. 5.10) is not.
3. Justify why Armstrong's definition of barycentric subdivision in §6.2 and tom Dieck's definition in Example 8.1.3 (for E finite) coincide.
4. Describe a Whitehead complex structure for the space $\mathbb{CP}^n$ in which there are only m-cells for m even.
5. (Whitney's umbrella). Consider the surface $W \subseteq \mathbb{R}^3$ given by the equation $x^2 - y^2 z = 0$
   (a) Sketch the surface W.
   (b) Descibe a Whitehead complex structure for W with one 0-cell, one 1-cell and two 2-cells.
   (c) Find a sequence of points x_i in one of the 2-cells so that $x_i \to x$ as $i \to \infty$ for some $x \in W$, but where there is a tangent vector v to the surface W at the point x that is not a limit of tangent vectors to the points x_i as $i \to \infty$.
6. Show that for any finite simplicial complex K with k-simplices only for $k \leq n$, the geometric realization $|K|$ can be realized as a polyhedron in $\mathbb{R}^{2n+1}$ (Hint: show that in $\mathbb{R}^{2n+1}$ it is possible to find arbitrarily large sets of points so that any subset of size at most $2n+2$ is geometrically independent, meaning that they form the vertices of a geometric $2n+1$-simplex).