Homology and Cohomology – Week 5 Exercises

Read the subsections of Hatcher's *Algebraic Topology* §2.1 entitled "Simplicial homology" and "Singular homology" (note, Hatcher likes to use a generalization of a simplicial complex called a Δ-complex. Take a look at the subsection with that title to see what the difference is. Then read tom Dieck's *Algebraic Topology* section 9.1.

Think about these exercises, and write up solutions to three of them. If you like you can also submit exercise solutions from last week.

1. (a) Solve Hatcher's exercise 2 on page 131.
   (b) Compute the simplicial homology of the Δ-complex you found that deformation retracts onto the Klein bottle.
   (c) Compute the simplicial homology of the Δ-complex you found that deformation retracts onto $\mathbb{RP}^2$.
2. Solve Hatcher's exercise 8 on page 131.
3. Let SimCom denote the category whose objects are simplicial complexes and whose morphisms are simplicial maps. That is, if $(V_1, S_1), (V_2, S_2)$ are simplicial complexes functions $f\colon V_1 \to V_2$ between the sets of vertices so that if σ is a simplex in S_1 then the image $f(\sigma)$ is a simplex in S_2. Let Chain denote the category of chain complexes over $\mathbb{Z}$. Show that simplicial homology defines a functor $\mathrm{SimCom} \to \mathrm{Chain})$ (you will need to say what the functor should do to a morphism).
4. Likewise, show that singular homology defines a functor from the category Top of topological spaces and continuous maps to Chain.
5. Compute the homology groups $\mathrm{H}_\bullet(X, A)$ where $X = S^1$ or $S^1 \times S^1$ and A is a finite subset (for instance, you could use the exact sequence tom Dieck gives as Theorem 9.1.3).