Homology and Cohomology – Week 8 Exercises

Read Section 3.1 of Hatcher's *Algebraic Topology* (pages 190–204. You could also take a look at the introduction to Section 3). Then take a look at the shorter exposition in tom Dieck's book, Section 17.5 (pages 416–419). Finally, for the tensor product, read through §10.4 of Dummit and Foote's *Abstract Algebra*.

Think about these exercises, and write up solutions to three of them. If you like you can also submit exercise solutions from previous weeks.

1. Hatcher exercise 1 on page 204.
2. Hatcher exercise 3 on page 204.
3. Compute $\mathrm{H}^\bullet(\mathbb{RP}^n; \mathbb{Z}/2)$ using the universal coefficient theorem. You can use the singular homology of $\mathbb{RP}^n$ which is given in Hatcher's book on page 144.
4. If R is a subring of S, prove that $S \otimes_R R^n \cong S^n$ for any natural number n.
5. If R is a ring, let R-mod denote the category of right R-modules. Prove that if S is a subring of R then $S \otimes_R -$ defines a functor R-mod $\to$ S-mod.
6. Solve Dummit and Foote problems 3–5 on page 375.
7. Take a look at tom Dieck's problem 1 on page 419. Prove that the Bockstein operator β is a natural transformation, and investigate the "typical and interesting case" that he describes.