Homology and Cohomology – Week 9 Exercises

Read Page 206–219 of Hatcher's *Algebraic Topology* Then take a look at Sections 11.8, 11.10 and 17.6 in tom Dieck's book.

Think about these exercises, and write up solutions to three of them. If you like you can also submit exercise solutions from previous weeks.

1. Hatcher's exercise 1 on page 228.
2. Compute $\mathrm{H}^\bullet(\Sigma_g;\mathbb{Z})$ as a ring, where Σ_g is the surface of genus g.
3. Hatcher's exercise 7 on page 229.
4. Hatcher's exercise 9 on page 229.
5. Prove that $S^n \times S^m$ is not homotopy equivalent to $S^n \vee S^m \vee S^{n+m}$.
6. Compute $\mathrm{H}^\bullet(\mathbb{CP}^2 \# \mathbb{CP}^2)$ as a ring (you might want to use a map to $\mathbb{CP}^2 \vee \mathbb{CP}^2$). Deduce that $\mathbb{CP}^2 \# \mathbb{CP}^2$ is not homotopy equivalent to $\mathbb{CP}^2 \times \mathbb{CP}^2$.