Math 797RM – Moduli Spaces in Geometry and Representation Theory – Syllabus

Spring 2021

Instructors

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Course plan

We'll meet on Monday, Wednesday and Friday through Zoom. You can use

Meeting ID: 946 1348 0790, Password: 78Vudsxk,

or the link https://umass-amherst.zoom.us/j/94613480790?pwd=T3hibXEwNHFmMHpOcmRKMC95TnV2dz09.

The course content will be adjusted based on participants' interests and background, but the provisional plan for the course is as follows:

- Week 1: Introduction to the course and to the functor of points approach to geometry.
- Week 2: First examples: Grassmannians and flag varieties, from several points of view.
- Week 3: Introduction to the Yang–Mills functional and instantons.
- Week 4: The main theorem & an introduction to quivers.
- Week 5–6: Quiver varieties, geometric invariant theory, and building towards the proof of a key theorem: comparing moduli spaces of instantons and certain quiver varieties. First student seminar talks.
- Week 7 and onwards: Further topics in moduli spaces based on participants' interests, and further student seminar talks.

What's expected of you

- The class will be discussion-based, and Zoom classes will include group discussion and collaborative activities.
- Starting around the third week, you will divide into small research groups based around topics you choose. For example, you might like to learn about things like:
 - The notion of a stack and its use in studying moduli problems.
 - Symplectic reduction & GIT (e.g., Kempf-Ness)
 - Higgs bundles and the Hitchin system.
 - Gauge theory and topology (e.g., Donaldson-Witten and Seiberg-Witten invariants).
 - Gauge theory and analysis (e.g., Uhlenbeck compactification, the twistor correspondence).
 - Moduli spaces of holomorphic vector bundles and stability conditions.
 - Hilbert schemes of surfaces.

These are just examples; we'll discuss other possibilities during class. The instructors will help point you to references in the literature, and you'll read and discuss them together with your group.

- Starting in week 5 of the semester, we will devote a meeting each week to one group, where the group will present about what they're learning. Ideally each group member will present at least once over the course of the semester.
- In the second half of the semester, you will individually write an expository article (at least 5 pages) about a topic from the material you studied and you will submit it to the *Journal of 797RM* for consideration. The instructors will act as the editorial board, will assign referees, and will give feedback on your report. Hence, each of you will anonymously review an article from another student and write a referee report, suggesting improvements. You will then edit your paper in response and resubmit.

We emphasize that we want this class to be collaborative and engaging, so we ask everyone to feel ready to share their thoughts and concerns kindly, thoughtfully, and honestly and we ask everyone to come ready to ask questions kindly, thoughtfully, and honestly. Feel free to talk with us (instructors and other students) about how the course is going and how it can evolve.

Accommodations

The University of Massachusetts Amherst is committed to making reasonable, effective and appropriate accommodations to meet the needs of students with disabilities and help create a barrier-free campus. If you are in need of accommodation for a documented disability, register with Disability Services to have an accommodation letter sent to your faculty. It is your responsibility to initiate these services and to communicate with faculty ahead of time to manage accommodations in a timely manner. For more information, consult the Disability Services website at http://www.umass.edu/disability/.

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