Math 131-H – Calculus 1 Honors **Name:** _____
Fall 2019
Final Exam (Practice 1)
12/16/19
Time Limit: 120 Minutes **Section Time (9:05 or 10:10):** _____

This exam contains 8 pages (including this cover page) and 7 problems.

You may not use your books, notes, or a calculator on this exam.

You are required to show your work on each problem on this exam: an incorrect answer supported by substantially correct calculations or explanations may still receive partial credit.

Problem	Points	Score
1	13	
2	10	
3	14	
4	15	
5	12	
6	12	
7	14	
Total:	90	

1. (13 points) (a) (3 points) State the product rule.

(b) (5 points) Calculate $\frac{\mathrm{d}}{\mathrm{d}x} 2x^5 \sin(x)$.

(c) (5 points) $\frac{\mathrm{d}}{\mathrm{d}x} 2x^5 \sin(x) \cos(x)$.

2. (10 points) (a) (5 points) Use the chain rule or implicit differentiation to show that $\frac{\mathrm{d}}{\mathrm{d}x}\log(x) = 1/x$. Here $\log(x)$ denotes the natural logarithm.

(b) (5 points) Find $\frac{\mathrm{d}}{\mathrm{d}x}\log(x^{-2})$.

3. (14 points) Consider the *hyperbola* with equation $2y^2 - x^2 = xy + 9$.

   (a) (7 points) Use implicit differentiation to find an equation for the slope of the tangent line to the curve.

   (b) (7 points) Find the two points on the curve where the tangent line is horizontal.

4. (15 points) Consider the function $f(x) = x^3 - 6x^2 + 9x$.

   (a) (5 points) Find the global maxima and minima of $f(x)$ on the interval $[0, 3]$.

   (b) (4 points) Find the roots of $f(x)$, i.e. the x-values where $f(x) = 0$.

   (c) (6 points) Sketch the graph of $f(x)$.

5. (12 points) Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1. \end{cases}$$

(a) (4 points) Is $f(x)$ continuous for all x? Why or why not?

(b) (4 points) Is $f(x)$ differentiable for all x? Why or why not?

(c) (4 points) Is $f(x)$ twice differentiable for all x? Why or why not?

6. (12 points) (a) (6 points) Evaluate $\lim_{x\to 0} \frac{\sin(2x)}{1-\cos(2x)}$.

(b) (6 points) Evaluate $\lim_{x\to \frac{\pi}{2}} \frac{\sin(2x)}{1-\cos(2x)}$.

7. (14 points) (a) (6 points) State the definition of the integral $\int_a^b f(x)\mathrm{d}x$ as a Riemann sum.

(b) (8 points) Compute $\int_0^1 x^2\mathrm{d}x$ using the Riemann sum definition. You may use the facts that $\sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}$, and $\sum_{k=0}^{n-1} k^2 = \frac{n(n-1)(2n-1)}{6}$.