

Math 131-H – Calculus 1 Honors

Name: _____

Fall 2019

Final Exam (Practice 2)

12/16/19

Time Limit: 120 Minutes

Section Time (9:05 or 10:10): _____

This exam contains 9 pages (including this cover page) and 7 problems.

You may not use your books, notes, or a calculator on this exam.

You are required to show your work on each problem on this exam: an incorrect answer supported by substantially correct calculations or explanations may still receive partial credit.

Problem	Points	Score
1	15	
2	11	
3	10	
4	14	
5	14	
6	14	
7	12	
Total:	90	

1. (15 points) Calculate $\frac{d}{dx}f(x)$ where $f(x)$ is each of the following functions.

(a) (5 points) $f(x) = \frac{\cos(2x)}{x^2}$.

(b) (5 points) $f(x) = \cos(2x^2)$.

(c) (5 points) $f(x) = \cos(\log(x^2))$, where $\log(x)$ is the natural logarithm.

2. (11 points) (a) (5 points) State the limit definition of the derivative.

(b) (6 points) Let $f(x) = x^{-2}$. Use the limit definition of the derivative and the limit laws to show that $\frac{d}{dx}f(x) = -2x^{-3}$.

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3. (10 points) Consider the *ellipse* described by the equation $9x^2 + 16y^2 = 25$.
- (a) (5 points) Use implicit differentiation to find an equation for the slope of the tangent line to the ellipse.
- (b) (2 points) Show that the point $(x, y) = (1, 1)$ lies on the curve.
- (c) (3 points) Find an equation for the tangent line to the curve at the point $(1, 1)$.

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4. (14 points) Consider the function $f(x) = 1 + 1/x - 1/x^2$.
- (a) (3 points) Find the roots of $f(x)$, i.e. the x -values where $f(x) = 0$.
- (b) (5 points) Find the critical points of $f(x)$, and determine whether they are maxima, minima or inflection points.

(c) (6 points) Sketch the graph of $f(x)$.

5. (14 points) (a) (4 points) State the definition of $\sinh(x)$ and $\cosh(x)$ in terms of exponential functions.

(b) (5 points) Use the exponential definition to show that $\cosh^2(x) - \sinh^2(x) = 1$ for all x .

(c) (5 points) Use the exponential definition to show that $\frac{d}{dx} \sinh(x) = \cosh(x)$.

6. (14 points) (a) (4 points) State L'Hôpital's theorem.

(b) (3 points) Is the limit $\lim_{x \rightarrow 0^+} \cos(x)^{1/x^2}$ an indeterminate form? If so, of which type?

(c) (7 points) Evaluate the limit $\lim_{x \rightarrow 0^+} \cos(x)^{1/x^2}$.

7. (12 points) (a) (4 points) Find the derivative of the function $F(x) = x \log(|x|) - x$, where $\log(x)$ is the natural logarithm.

(b) (3 points) Let $f(x) = \log(|x|)$. Where is $f(x)$ continuous?

(c) (5 points) Find all the possible antiderivatives of the function $f(x) = \log(|x|)$.