## Math 131-H - Homework 2 - Tangent Lines

## Due: in class on Tuesday October 1st.

1. Consider the function $f(x)=\frac{1}{4 a} x^{2}$ whose graph is a parabola with focus at the point $(0, a)$. The reflection property of the parabola says that the reflection of any vertical straight line in the parabola will pass through the focus.

(a) Find the $y$-intercept of the tangent line to the parabola at the point $\left(x_{0}, f\left(x_{0}\right)\right)$ (that is, the point where the tangent line meets the $y$ axis).
(b) Show that the triangle with vertices at $\left(x_{0}, f\left(x_{0}\right)\right)$, the focus of the parabola, and the $y$-intercept from (a), is isosceles.
(c) Hence show that the reflection property holds.
2. Let $\left(x_{0}, y_{0}\right)$ be a point in the plane. How many tangent lines to the hyperbola $y=1 / x$ pass through the point $\left(x_{0}, y_{0}\right)$ in the following cases:
(a) When $x_{0} y_{0}>1$ ?
(b) When $x_{0} y_{0}=1$ ?
(c) When $0<x_{0} y_{0}<1$ ?
(d) When $x_{0} y_{0}=0$, but $\left(x_{0}, y_{0}\right) \neq(0,0)$ ?
(e) When $\left(x_{0}, y_{0}\right)=(0,0)$ ?
(f) When $x_{0} y_{0}<0$ ?

In each case, find the equations of all the tangent lines through the point $\left(x_{0}, y_{0}\right)$.

