Math 131-H – Homework 3 – The Chain Rule

Due: in class on Thursday October 17th.

1. The n^{th} *Hermite polynomial* is $H_n(x)$ is the polynomial defined by

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n} e^{-x^2/2} = H_n(x) e^{-x^2/2}.$$

   (a) Compute $H_1(x), H_2(x), H_3(x)$ and $H_4(x)$.

   (b) Find a formula for $H_n(x)$ in terms of $H_{n-1}(x)$ and the derivative $H'_{n-1}(x)$.

2. The *Oval of Cassini* of eccentricity b is defined to be the curve consisting of all points

$$\{(x, y) \text{ in the plane} : \operatorname{dist}((x, y), (1, 0)) \times \operatorname{dist}((x, y), (-1, 0)) = b^2\}$$

   for each constant $b > 0$, where we have written $\operatorname{dist}((x_1, y_1), (x_2, y_2))$ to mean the distance from point (x_1, y_1) to point (x_2, y_2).

   (a) Show that the Oval of Cassini of eccentricity b is defined by the equation

$$(x^2 + y^2)^2 - 2(x^2 - y^2) + 1 = b^4.$$

   (b) Find an equation for the slope of the tangent line at a point on the curve in terms of x and y.

   (c) At which points (x_0, y_0) is the tangent line vertical? At which points is it horizontal? (Note: we did an example of this in class, specifically we worked this out for the lemniscate, which corresponds to $b = 1$.)

   (d) Sketch the Oval of Cassini when $b^2 = 0.5$, when $b^2 = 1.5$ and when $b^2 = 3$. Mark the points you found in part (c) on your graphs.