Math 131-H – Homework 5 – Curve Sketching

Due: in class on Thursday November 14th.

1. Consider the function $f(x) = 2x^3 - bx^2 + cx$, where b and c are constants.

   (a) Suppose the graph $y = f(x)$ has one minimum at the point $(x, y) = (p, f(p))$, and one maximum at $(x, y) = (q, f(q))$. Find p and q in terms of b and c. What condition must b and c satisfy for the curve to have one minimum and one maximum?

   (b) For which x is the graph $y = f(x)$ concave upwards? For which x is it concave downwards?

   (c) Sketch the graph $y = f(x)$, marking the points $(p, f(p)), (q, f(q))$, and any points where the curve changes concavity.

2. Consider the curve given by graphing the function $f(x) = \frac{x}{\sqrt{x^2-2x+a}}$, where a is a constant.

   (a) If $a > 1$, where is $f(x)$ defined? What if $a \leq 1$?

   (b) Show that, if $a > 1$, then the graph $y = f(x)$ has exactly one critical point.

   (c) Compute $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

   (d) Sketch a graph of $y = f(x)$ when $a = 2$.

   (e) Sketch a graph of $y = f(x)$ when $a = 1$ (Hint: don't forget that $\sqrt{u^2} = |u|$, not just u).