Math 131-H – Homework 6 – Integration

Due: in class on Tuesday December 3rd.

1. Fix $a < b$ and any integrable function $f(x)$. For which value of c is the expression $\int_a^b (f(x) - c)^2$ smallest? (Hint: you don't need to use the fundamental theorem of calculus: just use what you know about finding maxima and minima)

2. Let $f(x) = e^x$, on the interval $[0, 1]$

   (a) Apply the mean value theorem to the function $f(x)$ over the interval $[0, x]$ to show that the curve $y = e^x$ lies between the lines $y = 1 + x$ and $y = 1 + 3x$ whenever x is between 0 and 1.

   (b) Use this result to show that $1 < \int_0^1 e^x \mathrm{d}x < 2$ without evaluating the integral.

3. Let $f(x) = x^3 + x^2$ on the interval $[0, 2]$.

   (a) Show that $\sum_{k=0}^{n-1} k^2 = \frac{n(n-1)(2n-1)}{6}$ (Hint: expand out $(k + 1)^3 - k^3$, then take the sum from $k = 0$ to $n - 1$).

   (b) Show that $\sum_{k=0}^{n-1} k^3 = \frac{n^2(n-1)^2}{4}$ (Hint: expand out $(k + 1)^4 - k^4$, then take the sum. You'll need to use the sum from (a)).

   (c) Calculate $\int_0^2 f(x)\mathrm{d}x$ by computing and simplifying the Riemann sums A_n for each n, then taking the limit as $n \to \infty$.