

Math 131-H – Calculus 1 Honors

Name: _____

Fall 2019

Midterm 2 (Practice)

11/3/19

Time Limit: 60 Minutes

Section Time (9:05 or 10:10): _____

This exam contains 5 pages (including this cover page) and 4 problems.

You may not use your books, notes, or a calculator on this exam.

You are required to show your work on each problem on this exam: an incorrect answer supported by substantially correct calculations or explanations may still receive partial credit.

Problem	Points	Score
1	10	
2	14	
3	11	
4	15	
Total:	50	

1. (10 points) Use the chain rule to differentiate the following functions.

(a) (5 points) $f(x) = \frac{1}{\sqrt{x^3+2x}}$.

(b) (5 points) $g(x) = \log(\sec(x))$.

2. (14 points) Recall that $\cosh^2(x) - \sinh^2(x) = 1$ for all x , and therefore that $1 - \tanh^2(x) = \operatorname{sech}^2(x)$.

(a) (3 points) Use this fact to show that $\tanh^2(\operatorname{arcsech}(x)) = 1 - x^2$.

(b) (4 points) Use the quotient rule to find $\frac{d}{dx}\operatorname{sech}(x)$.

(c) (7 points) Use implicit differentiation to show that

$$\frac{d}{dx}\operatorname{arcsech}(x) = -\frac{1}{x\sqrt{1-x^2}}.$$

3. (11 points) *Tschirnhausen's Cubic* is the curve defined by the equation $3y^2 = x^2(1 - x)$.

(a) (5 points) Use implicit differentiation to find the slope $\frac{dy}{dx}$ of a tangent line to the Tschirnhausen's cubic.

(b) (3 points) Show that the point $(x, y) = (1/2, -1/\sqrt{24})$ lies on the curve.

(c) (3 points) Find the slope of the tangent line to the curve at the point $(x, y) = (1/2, -1/\sqrt{24})$.

4. (15 points) Consider the function

$$f(x) = x^2 - 2x - 4\log(|x|)$$

where $\log(x)$ is the natural logarithm.

- (a) (4 points) Find the critical points of $f(x)$.

- (b) (5 points) Using the second derivative test, or otherwise, determine whether the critical points are maxima, minima or inflection points.

- (c) (6 points) $f(x) = 0$ when x is approximately 0.8 and 3.4. Use this, and your answers to the first two parts, to sketch the graph $y = f(x)$.