NORTHWESTERN MASTERCLASS
HEEGAARD FLOER LECTURE SERIES
HOMEWORK 1

(1) Recall that the Morse lemma states that if p is a critical point of a Morse function f then there is a neighborhood $U \ni p$ and coordinates $x_1, \dots, x_n$ on U so that

$$f(x_1, \dots, x_n) = -x_1^2 - \cdots - x_k^2 + x_{k+1}^2 + \cdots + x_n^2.$$

Prove that the integer k is well-defined. That is, if $y_1, \dots, y_n$ is another choice of coordinates on U so that

$$f(y_1, \dots, y_n) = -y_1^2 - \cdots - y_\ell^2 + y_{\ell+1}^2 + \cdots + y_n^2$$

then $k = \ell$.

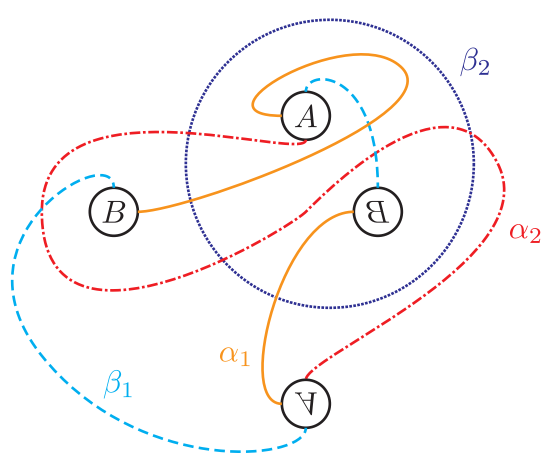
(2) Prove: if v is a gradient-like vector field for f and $\gamma(t)$ is a flow-line of v then $\lim_{t\to\infty} \gamma(t)$ is a critical point of f, as is $\lim_{t\to-\infty} \gamma(t)$.

(3) Recall that

$$U(p) = \{x \in M \mid \lim_{t\to-\infty} \gamma_x(t) = p\}$$
$$D(p) = \{x \in M \mid \lim_{t\to+\infty} \gamma_x(t) = p\},$$

denote the ascending and descending disks of x. Prove that these are, in fact, (open) disks.

(4) Consider the following Heegaard diagram for a 3-manifold Y:



(The circles with letters in them denote handles. Delete the interiors of these circles, and glue them together according to the labels.)

Compute $H_1(Y)$, $H_2(Y)$ and $\pi_1(Y)$. (There are several different ways to approach this.)

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